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APPLIED MECHANICS

ПРИКЛАДНА МЕХАНІКА

Рекомендовано Міністерством освіти і науки України
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1 DYNAMICS of MECHANISMS

1.1 Structure of mechanisms

1.1.1 Basic elements of mechanisms

Let's consider the mechanism of a drive. The mechanical drive is a device for transmitting of motion from the motor to the working machine. In figure 1.1 the drive of the rolling mill is shown.

Motion is transmitted from the alternating current motor (1) to the rollers of the rolling mill (8) that process hot ingots (10). The direct current motor (6) is supplied by generator (5). The reduction gear (7) reduces speed of rotation. The fly-wheel (4) smoothes the load on the motor (1) and makes a uniform motion of a drive. The drive provides necessary speed, torsion moment and no uniform rotation of rollers (8).

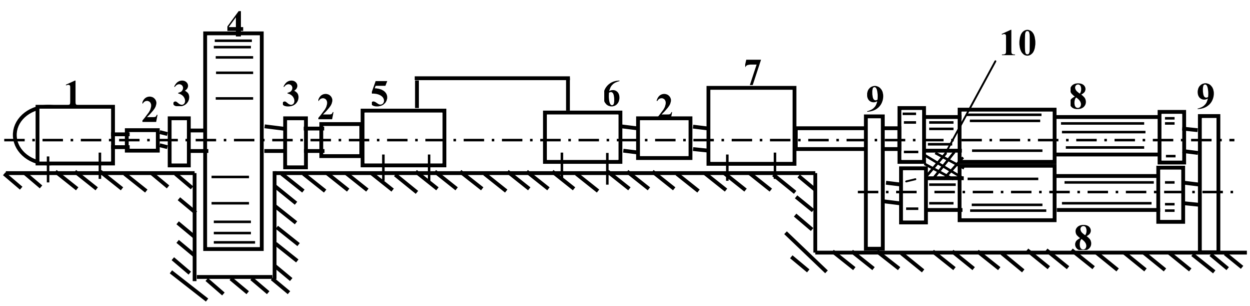

Fig. 1.1 – The drive of the rolling mill
1- alternating current motor; 2- couplings; 3- support of fly-wheel;
4- fly-wheel; 5- direct current generator; 6-direct current motor;
7- reduction gear; 8- rollers of rolling mill; 9- supports; 10- hot ingot

Calculation of a drive provides the solution of such problems.

The drive should provide necessary speed of a final link. The usual drive has an asynchronous motor with frequency of rotation 750, 1500, 3000 *r.p.m*. The initial shaft of the working machine has frequency of rotation from 20 to 150 *r.p.m*. Thus, decrease in speed is one of the main problems of the drive.

Perfection of a design is characterised by a minimum of energy losses and maximum of mechanical efficiency. The calculation of the latter is one of drive dynamics problems.

The forces of the motor and the working machine are changing, thus the drive moves non-uniformly. Normal work of the machine is possible at small, allowable non-uniformity of motion. Usually non-uniformity of motion is reduced by setting-up of additional masses in a drive.

Thus, calculation of masses and forces in a drive is one of the main problems of drive designing. Two methods of calculation are used. The first method calculates dynamics of the mechanism with rigid and elastic parts. The second method provides calculation of a resonance in a drive and its prevention.

The design of the drive requires its structural analysis: determination of the links, kinematic pairs and degree of freedom.

1.1.2 Structural analysis of a mechanism

A mechanism is a set of bodies intended for transformation of a motion. The mechanism consists of links and kinematic pairs.

The link is a hard body or an assemblage of motionlessly connected hard bodies. The mechanism has mobile and immobile links. The immobile link is called a support. The link is called driver, if it is connected to the motor.

The kinematic pair is a mobile connection of two links. The pairs are divided into classes. The class is determined by a number of connections in pair (Fig. 1.2). Links and kinematic pairs form a kinematic chain.

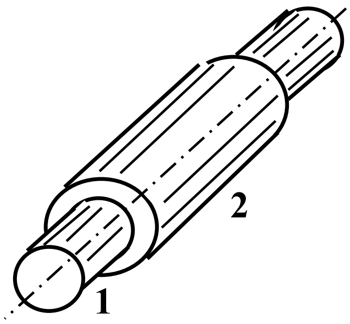

Fig. 1.2 – Kinematic pair of fourth class

Degree of mechanism freedom is determined by a number of leading links. A degree of flat mechanism freedom is determined by formula (1.1)

$$W = 3n - 2p_5 - p_4, \qquad (1.1)$$

where n is a number of mobile links; p_5 is a number of the 5-th class kinematic pairs ; p_4 is a number of the 4-th class kinematic pairs.

The mechanism (Fig.1.3) has $n = 3$ (links 1,2,3), $p_5 = 4$ (pairs A,B,C,D), $p_4 = 0$. For such mechanism we have

$$W = 3n - 2p_5 - p_4 = 3\cdot 3 - 2\cdot 4 - 0 = 1,$$

That means, that the degree of freedom is equal to 1 and the number of leading links is also equal to one. There may be several driving parts in the mechanism. Such mechanism is refered to as differential as it constitutes movements caused by driving links.

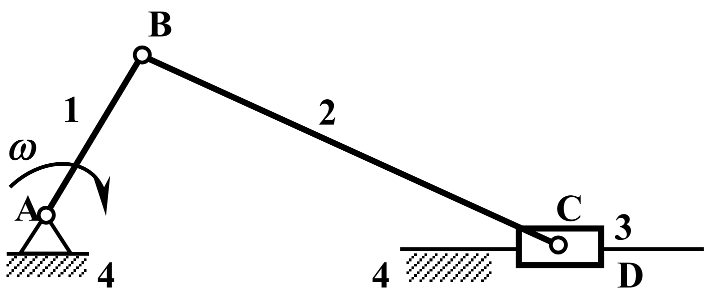

Fig. 1.3 – Crank-slide drive

The degree of freedom of a spatial mechanism equals to

$$W = 6n - 5p_5 - 4p_4 - 3p_3 - 2p_2 - p_1, \qquad (1.2)$$

where n is a number of mobile links; p_1, p_2,..., p_5 is a number of kinematic pairs of corresponding classes.

The formulas (1.1), (1.2) are correct if kinematic pairs have independent connections. The mechanism may have surplus connections and superfluous mobilities.

1.1.3 Surplus connections and superfluous mobilities of a mechanism

The connections and the mobilities, which do not influence the motion of a mechanism are called surplus connections and superfluous mobilities correspondingly. Let's consider the flat and spatial four-bar mechanism (Fig.1.4). The degree of freedom of a four-bar flat mechanism (Fig.1.4*a*) is equal to

$$W = 3n - 2p_5 - p_4, \quad \text{where } n = 3,\ p_5 = 4,\ p_4 = 0.$$

In that case the degree of freedom is equal to W=1.

The degree of freedom of a four-bar spatial mechanism (Fig.1.4*a*) is equal to

$$W = 6n - 5p_5 - 4p_4 - 3p_3 - 2p_2 - p_1; \quad \text{where} \quad n = 3,\ p_5 = 4,\ p_4 = p_3 = p_2 = p_1 = 0 .$$

The degree of freedom is equal to W=-2 and there are three surplus connections. Let's remove the surplus connections by pairs class decrease (Fig.1.4*b*). Then a new spatial mechanism has n =3, p_5 =2, p_4 =1, p_3 =1, p_2 =p_1 =0 and its degree of freedom is W=1. Then we can lower the class of pair B(4) to the third class. In that case (Fig. 1.4*c*) the mechanism will have two degrees of freedom – motion of the first link and a local mobility of the second link.

The mechanisms with the surplus connections are used for transmitting the large loads (presses, internal combustion engines, hammers, etc.). The necessary class of kinematic pairs in machine is formed by the running-in. The mechanisms with the superfluous mobilities are used for transmitting the precise motion (control mechanisms) or for compensating errors in mechanisms manufacturing.

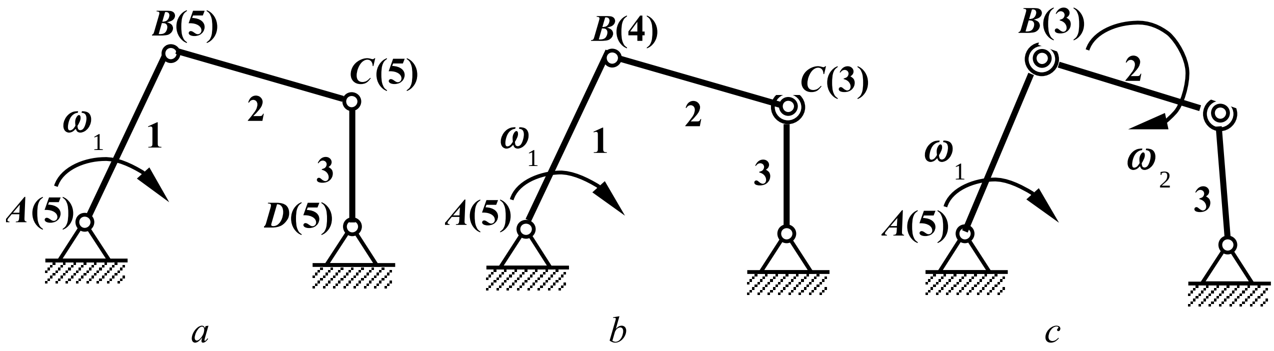

Fig 1.4 – Spatial four-bar mechanism
a – with surplus connections, *b* – without surplus connections,
c – with local mobility of second link

1.1.4 Assur's groups

According to I.Artobolevsky's classification [1] based on L.Assur's idea , it is possible to construct any mechanism by successive connection of an initial entrance link and Assur's group.

Assur's group is a kinematic chain, which has a degree of freedom equal to zero, if the free kinematic pairs are connected to a support. Assur's group has only 5-th class kinematic pairs. In that case the degree of freedom of Assur's group is equal to

$$W = 3n - 2p_5 = 0.$$

Assur's group consists of the conjugate number of links and the number of kinematic pairs divisible by three. For example, the mechanism of a power press (Fig. 1.5) has the initial link (1) and Assur's group of second (links 2, 3) class.

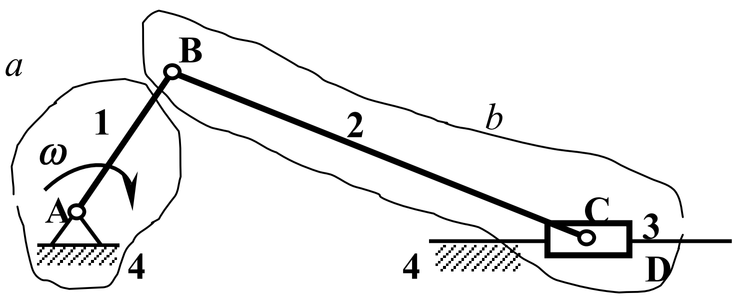

Fig. 1.5 – The mechanism of press
a – entrance link; *b* – Assur's group of second class

1.2 Kinematics of a mechanism

In kinematics the methods of determining the motion of a mechanism are studied. Two methods are usually practiced: analytical and graphical. In that chapter we will describe the methods of analysis of the ordinary gears and satellite gears.

1.2.1 Ratio of a toothed gear

The simple gear has two links, two lower and one higher kinematic pairs. The compound mechanism consists of several simple toothed gearings. The toothed gear can have cylindrical (axes of wheels are parallel), conical (axes of wheels are intersected) and hyperbolic (axes of wheels are crossed) wheels (Fig. 1.6).

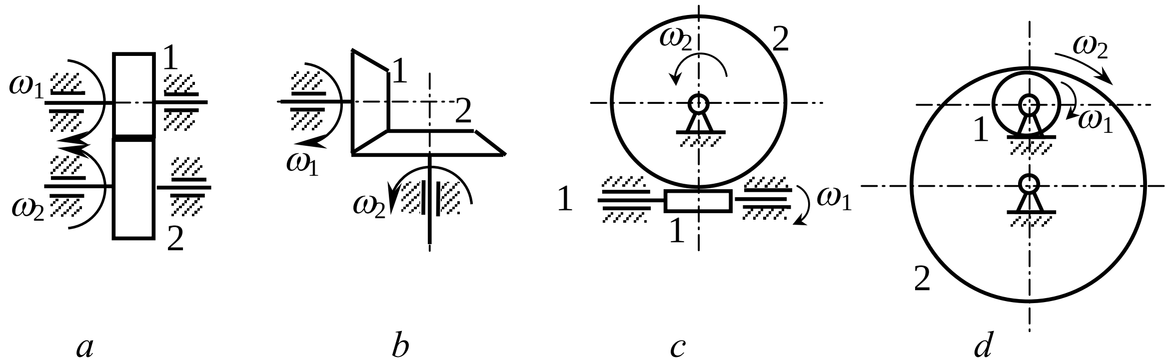

Fig. 1.6 – The cylindrical (*a, d*), conical (*b*), and hyperbolic (*c*) toothed gears

The toothed gear mechanisms can have external and internal gearing. The wheels of the external gearing rotate in different directions (Fig. 1.6*a*), the others rotate in same direction (Fig. 1.6*d*). The toothed gear has driving (1) and driven (2) wheels. The gears can be ordinary and satellite**.** The ordinary gears have motionless axes of the toothed wheels, the others have mobile axes.

The basic characteristic of a gear is a gear ratio. The gear ratio is denoted as U. The gear ratio is ratio of angular velocity of the driving wheel to the angular velocity of the driven wheel $U_{12} = \omega_1 / \omega_2$ (Fig. 1.6*a*). If $\omega = \pi \cdot n / 30$, where n is the speed in *r. p. m.*, the gear ratio is equal to

$$U_{12} = \frac{\omega_1}{\omega_2} = \pm \frac{n_1}{n_2}. \qquad (1.3)$$

In the formula (1.3) sign "plus" indicates the same direction of rotation for the drive and driven wheels and "minus" indicates different directions of rotation.

In conical gears the direction of rotation is determined according to the rule of arrows (Fig. 1.6*b*).

For spur, bevel gears the velocity in contact point (Fig.1.6*a*, 1.6*b*) of wheels is equal to

$$V_a = \omega_1 \cdot r_1 = \omega_2 \cdot r_2, \qquad (1.4)$$

where ω_1, ω_2 are the angular velocities of the wheels; r_1, r_2 are the radii of wheels.

Then from the equations (1.3) and (1.4) we obtain

$$U_{12} = \frac{\omega_1}{\omega_2} = \pm\frac{n_1}{n_2} = \pm\frac{r_2}{r_1}.$$

If the numbers of teeth are z_1, z_2 and a pitch of teeth is p, then the lengths of the circumferences of wheels are equal to

$$2\pi r_1 = pz_1, \quad 2\pi r_2 = pz_2$$

and the gear ratio is the following

$$U_{12} = \frac{\omega_1}{\omega_2} = \pm\frac{n_1}{n_2} = \pm\frac{r_2}{r_1} = \pm\frac{z_2}{z_1}. \qquad (1.5)$$

In that formula we have "plus" for the internal gearing and "minus" for the external gearing, because external gearing changes the direction of wheels rotation.

1.2.2 Gear ratio of an ordinary gear transmission

To obtain large gear ratio several simple gear mechanisms are connected [1]. Such connection is called a multistage one. The gear mechanism with fixed axes is called an ordinary one (Fig. 1.7).

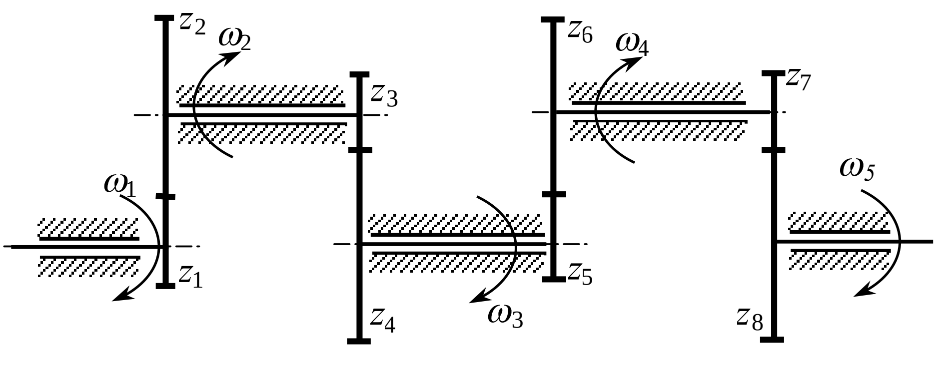

Fig. 1.7 – The multistage gear mechanism